Detecting Symmetries of All Cardinalities With Application to Musical 12-Tone Rows

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Abstract

Popularized by Arnold Schoenberg in the mid-20th century, the method of twelve-tone composition produces musical compositions based on one or more orderings of the equal-tempered chromatic scale. The work of twelve-tone composers is famously challenging to traditional Western tonal and structural sensibilities; even so, group theoretic approaches have determined that 10% of certain composers' works contain a highly unusual classical symmetry of music. We extend this result by revealing many symmetries that were previously undetected in the works of Schoenberg, Webern, and Berg. Our approach is computational rather than group theoretic, scanning each composition for symmetries of many different cardinalities. Thus, we capture partial symmetries that would be overlooked by more formal means. Moreover, our methods are applicable beyond the narrow scope of twelve-tone composition. We achieve our results by first extending the group-theoretic notion of symmetry to encompass shorter motives that may be repeated and reprised in a given composition, and then comparing the incidence of these symmetries between the work of composers and the space of all possible 12-tone rows. We present four candidate hierarchies of symmetry and show that in each model, between 75% and 95% of actual compositions contained high levels of internal symmetry.

Introduction

The Viennese composers Arnold Schoenberg, Anton Webern, and Alban Berg produced a combination of 86 twelve-tone compositions in the early-to-mid 20th century. Each of these compositions is constructed from some permutation of the twelve pitch classes of the equal-tempered chromatic scale, which then guides the order of notes in the melody and harmonies. Figure 1 shows musical notation for one such permutation of the pitch classes that was selected by Schoenberg. Each such permutation is called a *tone row*, and we will take the *Viennese tone rows* to mean those that underlie the compositions of Schoenberg, Webern, and Berg, the principal members of the Second Viennese School.

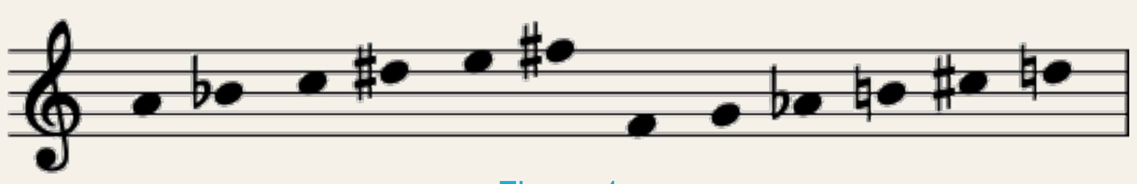

Figure 1

Using Hauer's arrangement of the pitch classes at the hour marks of an analog clock diagram, each tone row can be visualized as a directed graph that visits each vertex once, and the set of these directed graphs is in bijection with the set of tone rows. Figure 2 shows clock diagrams for the circle of fifths and an all-interval row.

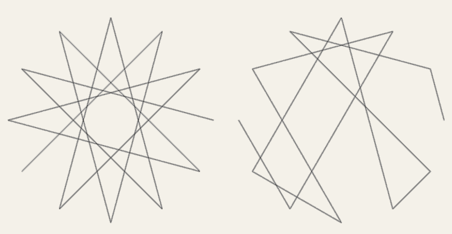

Figure 2

Symmetry in 12-Tone Rows

Hunter and von Hippel posed and resolved the following question: are there more symmetric Viennese tone rows than would be expected by chance alone? By the numbers, symmetric rows comprise just 5% of Schoenberg's works, 19% of Webern's, and 9% of Berg's. However, symmetric rows are *exceedingly* rare among all row classes: just 0.13% of row classes possess symmetric clock diagrams. Hunter and von Hippel made this idea rigorous by using a hypergeometric test to conclude that these composers showed a statistically significant preference for symmetry, whether intentionally or not. Figure 3 shows clock diagrams of Webern's tone rows with the four symmetrical ones set apart.

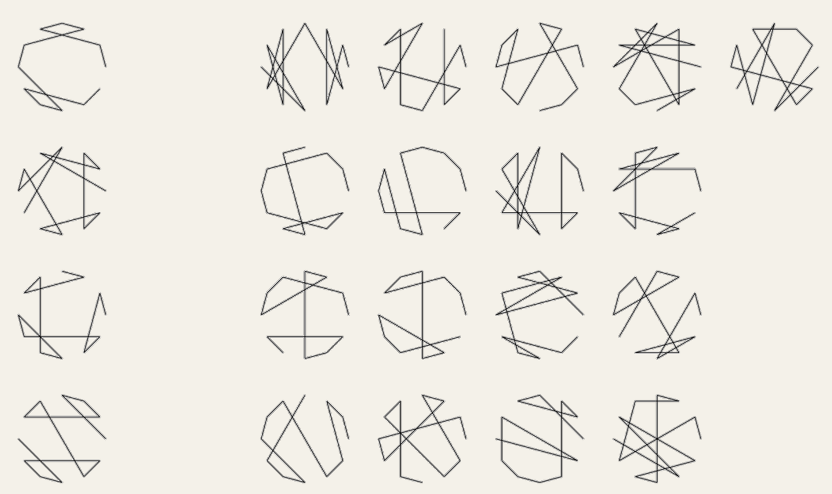

Figure 3

An Open Question: "Partially Derived" Rows

The combinatorial study of tone rows has led to the enumeration of rows with various traditionally studied properties as well as measurements of "tonalness" and other harmonic properties. Meanwhile on the group-theoretic side, Fripertinger and Lackner's group action has helped enumerate the derived rows, a much richer set than the fully symmetric rows of Hunter and von Hippel. Missing from the literature is a combinatorial attack on the *definition* of symmetry in tone rows: just as Yust extended von Hippel and Huron's study of tonal fit to pairs of tonal, atonal, and mixed dimensions, this paper uses combinatorial means to generalize the concept of derived row, capturing nuances that are impractical to detect with group theory. For example, if we permute just the last two notes of a derived row, the resulting tone row will most likely not possess any group theoretic symmetries. The rigidity of group-theoretic symmetries causes this type of partial symmetry to be overlooked. In this work, we introduce a combinatorial alternative to the group-theoretic definitions of symmetry, scanning each tone row for partial symmetries of all cardinalities. Our approach results in a complete catalog of recurring motives within each tone row, from which we argue that the vast majority of the Viennese tone rows contain unusually high levels of symmetry.

Detecting Internal Symmetry

For the purpose of this work, a *motive* within a tone row is any consecutive sequence of notes that recurs at least once within the same row (transposed and possibly inverted or in retrograde). For example, the row (0, 1, 10, 5, 2, 3, 6, 7, 8, 11, 4, 9) contains the motive (0, 1, 10, 5) which repeats transposed under retrograde inverse as (10, 5, 2, 3), as shown in the top line of Figure 4. This particular row also contains the motive (3, 6, 7, 8) and its transposed retrograde inverse (6, 7, 8, 11). Motives can be as short as two notes or as long as twelve; whereas motives of length 2 occur whenever the same musical interval (or its inverse) is reused in a tone row, motives of length 12 are only found in the rare palindromic rows.

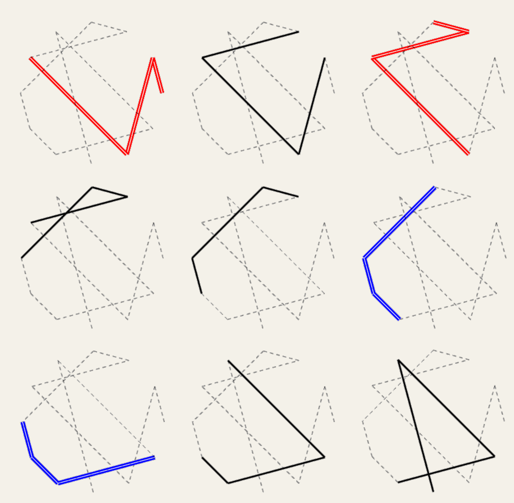

Figure 4

For each row class, we note the most repetitive motive of each length from 2 to 12. Figure 5 shows a row class whose most-repeated motives of length 2, 3, and 4 have multiplicities of 3, 4, and 2, respectively. This particular row class has no motives of length 5 or greater, so we assign it an overall *symmetry score* of (3, 4, 2, 1, 1, 1, 1, 1, 1, 1, 1) representing the repetitiousness of its motives of length 2 to 12.

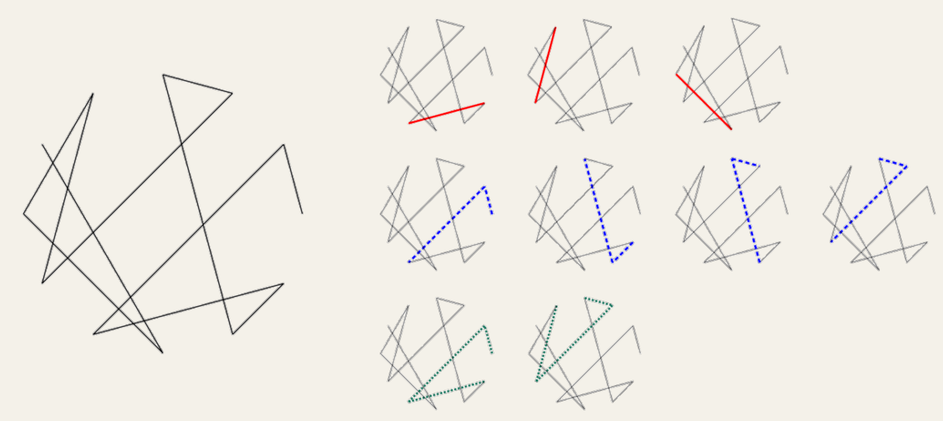

Figure 5

Identifying "High Symmetry"

The tone row *symmetry score* imparts a partial ordering to the set of tone rows, allowing us to gauge whether composers chose tone rows with unusually high internal symmetry. We offer four models here that generalize the literature in various competing ways:

- Reverse-Lexicographic Order
- Lattice Rank
- Partial Order Solo Rank
- Partial Order Cohort Rank

Four Competing Models Coincide

We first consider the reverse-lexicographic total order on the set of symmetry scores of tone rows. This yields a natural definition of low and high symmetry: for each length *d*, the valid splits are the locations in the total order where the *d*-symmetry score increments. Table 1 shows a selection of significant results for the three composers, most of which remain significant under a multiple testing correction (* : $p < 0.05$, ** : $p < 0.01$, *** : $p < 0.001$).

Schoenberg	All	Webern	All	Berg	All
76.19%**	45.50%	90.48%***	39.15%	–	–
50.00%**	23.62%	57.14%*	23.68%	47.83%	28.84%

Table 1

We then use a lattice grading to rank symmetry scores by their sum of components; here, the natural divisions of low and high symmetry are determined by the rank function with 54 distinct levels. Table 2 shows that the Lattice Rank model obtains remarkably similar results to the RLEX model in spite of being agnostic to motive length.

Schoenberg	All	Webern	All	Berg	All
83.33%	66.07%	95.24%	66.07%	–	–
47.62%**	20.87%	71.43%***	20.87%	56.52%	35.04%

Table 2

Lastly, we consider two models based solely on the relative counts of ascendants and descendants in the partial order. These models have the advantage that they make no value judgments on the relative importance of different motives. Figure 6 shows the two partially ordered sets of symmetry scores that arise in this work. Table 3 shows the results of the Partial Order Rank models (they coincide).

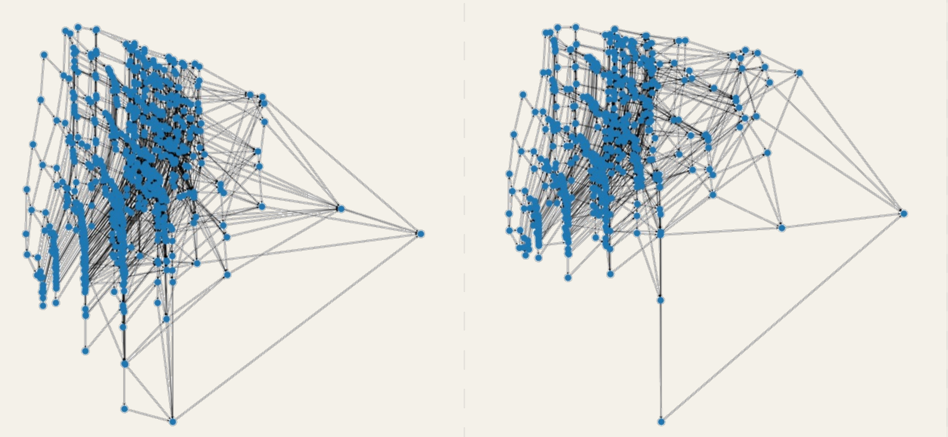

Figure 6

Schoenberg	All	Webern	All	Berg	All
83.33%	65.87%	95.24%	45.35%	–	–
52.38%	26.23%	66.67%	16.03%	52.17%	29.42%

Table 3

Conclusion

We found that 76%-83% of Schoenberg's corpus and 90%-95% of Webern's corpus contained significant levels of symmetry (up from 5% and 19%, respectively), even after applying a conservative Bonferroni correction for multiple testing. Berg's case involves a much smaller space of row classes which limited our study of his corpus to an exploratory basis (48%-57%, up from 9%).

Selected References

1. P. T. von Hippel and D. Huron, "Tonal and 'Anti-Tonal' Cognitive Structure in Viennese Twelve-Tone Rows"
2. D. J. Hunter and P. T. von Hippel, "How Rare is Symmetry in Musical 12-Tone Rows?"
3. H. Fripertinger and P. Lackner, "Tone rows and tropes"
4. M. Gotham and J. Yust, "Serial Analysis: A Digital Library of Rows in the Repertoire and their Properties"

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